AD-766 137

EFFECTS OF TURBULENCE INSTABILITIES ON LASER PROPAGATION (PHASE II)

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RCA Laboratories

Prepared for:

Advanced Research Projects Agency

August 1973

AD 766137

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ON LASER PROPAGATION
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RCA LABORATORIES

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SEP 5 1973

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ROME AIR DEVELOPMENT CENTER
AIR FORCE SYSTEMS COMMAND
GRIFFISS AIR FORCE BASE, NEW YORK

45

DOCUMENT CONTROL DATA · R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1 ORIGINATING ACTIVITY (Corporate author)	2a. REPORT SECURITY CLASSIFICATION
RCA Laboratories Princeton, NJ 08540	Unclassified
	2b. GROUP N/A

3 REPORT TITLE

EFFECTS OF TURBULENCE INSTABILITIES ON LASER PROPAGATION (PHASE II)

4 DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

Technical Report   (April 1973 to August 15, 1973)

5 AUTHOR(S) *(First name, middle initial, last name)*

David A. de Wolf

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
AUGUST 1973	45	11

8a. CONTRACT OR GRANT NO. F30602-72-C-0486	9a. ORIGINATOR'S REPORT NUMBER(S)
b. PROJECT NO. ARPA Order No. 1279	PRRL-73-CR-47
c. Program Code 1E20	9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*
d.	

10. DISTRIBUTION STATEMENT

Approved for public release; distribution unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
Monitored by: Capt. Darryl P. Greenwood, (315)330-3147, RADC (OCSE) GAFB, N.Y. 13441	Defense Advanced Research Projects Agency Washington, D.C. 20301

13. ABSTRACT

This report on the behavior of laser beams in turbulent air extends results summarized in a previous report (RADC-TR-73-162). It is shown that amplitude scintillation is unimportant in focused beams and is dominant in collimated and diverging beams. The limitations of an existing numerical procedure for computing these are investigated. Intermittency of turbulence is redefined in terms of large-scale variations in the refractive-index structure constant and the dependence upon averaging time is determined. It is concluded that intermittency is essentially an extra randomness during operation of the laser for short periods (of several minutes or less). Significant fluctuation rates are defined for angle of arrival, phase, and amplitude. They differ greatly.

DD FORM 1473 (1 NOV 55)

1a

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Wave propagation						
Turbulent atmosphere, air						
Laser beam, focused						
Optics, atmospheric						
Irradiance fluctuations						

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DAVID A. de WOLF

CONTRACTOR: RCA LABORATORIES
CONTRACT NUMBER: F30602-72-C-0486
EFFECTIVE DATE OF CONTRACT: 15 FEBRUARY 1973
CONTRACT EXPIRATION DATE: 15 AUGUST 1973
AMOUNT OF CONTRACT: $24,446.00
PROGRAM CODE NUMBER: 1E20

PRINCIPAL INVESTIGATOR: DR. DAVID A. de WOLF
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PHONE: 315 330-3147

Approved for public release; distribution unlimited.

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PUBLICATION REVIEW

This technical report has been reviewed and is approved

Darryl P. Greenwood
RADC Project Engineer

FOREWORD

This Technical Report was prepared by RCA Laboratories, Princeton, New Jersey, under Contract No. F30602-72-C-0486. It describes work performed from April 1973 to August 15 1973 in the Communications Research Laboratory, Dr. K. H. Powers, Director. The principal investigator and project scientist was Dr. D. A. de Wolf.

The report was submitted by the author in August 1973. Submission of the report does not constitute Air Force approval of the report's findings or conclusions. It is submitted only for the exchange and stimulation of ideas.

The Air Force Program Monitor is Capt. Darryl P. Greenwood.

SUMMARY AND ABSTRACT

This Final Report of Contract No. F30602-72-C-0486 regarding the behavior of laser beams in turbulent air extends results summarized in a previous report (RADC-TR-73-162). It is shown that amplitude scintillation is unimportant in focused beams, and dominant in collimated and diverging beams. The limitations of an existing numerical procedure for computing these are investigated. Intermittency of turbulence is redefined in terms of large-scale variations in the refractive-index structure constant and the dependence upon averaging time is determined. It is concluded that intermittency is essentially an extra randomness during operation of the laser for short periods (of several minutes or less). Significant fluctuation rates are defined for angle of arrival, phase, and amplitude. They differ greatly.

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1. INTRODUCTION

This is the Final Report of the RCA effort on the effect of turbulence instabilities on laser propagation under contract no. F30602-72-C-0486 to Rome Air Development Center. Previous reports under this and under the preceding effort (contract No. F30602-71-C-0356) are summarized [1] in the list of references, and they will be referred to as TRI to TRVIII in chronological order.

Aside from previously reported developments in TRI-TRVII, also summarized in TRVIII, this work has led to the following developments: In section 3, a generalization of the Pythagorean sum of focal-spot area and induced broadening by turbulent air is given for any image plane of a gaussian beam (diverging, collimated, or focusing). We prove in section 4 that amplitude scintillation in the focal plane of a focused beam is negligible, and therefore restrict the analytical development of laser-beam irradiance to the Rytov approximation in that case. Except for exceptional situations, the preceding saturation-regime calculations for plane and spherical waves suffice for non-focused beams. Herrmann and Bradley's numerical procedure is investigated in section 5 and it is shown that this procedure cannot predict large amplitude fluctuations. The problem of intermittency is discussed in section 6. The power spectrum of C_n^2 at low wave numbers is its determining factor, and we conclude that intermittency must be regarded as an unpredictable random effect governed by long-term statistics in view of the fact that the above-mentioned power spectrum is highly specific to terrain and local weather conditions. The idea that the angle-of-arrival spectrum can yield microscale information unfortunately must be abandoned (section 7). However, in section 8 we show that important fluctuation rates of angle of arrival, amplitude, and phase can be defined and computed. Section 2 is an introduction to the problems defined in the whole effort, and to the method of solution: it serves as an aid to the reader who does not wish to immerse himself in all the details of previous reports and work.

2. INTRODUCTION TO THE LASER-BEAM PROPAGATION PROBLEM

The work of the contract effort of which this is the Final Report is centered around the problem of forming an image in the plane $z = L$ of a laser beam originating at $z = 0$. The beam must travel through (possibly turbulent) air. One might distinguish between three aspects of the problem:

(i) The optics of imaging a source at $z = 0$ in the plane $z = L$ in the idealized case of free space between source and image. Thus, one might consider what optical functions (e.g., mutual transfer function, point spread function, etc.) give optimal information about the source or the imaged object.

(ii) The effect upon the imaging process of the air through which the beam travels from source to image plane. Air can absorb energy from laser beams; aerosols and other particulate matter can scatter energy out of the beam, and inhomogeneities in the gaseous properties (e.g., in temperature, density, water-vapor content) can give rise to deflection of rays. All of these effects influence the shape of the image.

(iii) The nature of the source or of the object to be imaged. In our work, the source is nearly always the output of a laser, and hence it is the shape of the transmitted laser beam that plays the role of a separate input to the problem.

The distinction has been overemphasized. Figure 1 illustrates the most important aspect of the problem: the interaction of light rays with air. A host of effects occurs even when the intensity is not extremely high (we have not sketched in an equally large host of other effects such as air breakdown which occur at extremely high intensities). It is important to realize, however, that the beam quality mentioned in (iii), i.e., whether one uses focused beams, or whether the beam is gaussian with or without truncation, has a definite effect upon the beam quality. For example, a highly focused beam will create higher intensities at the focal spot, and one might well expect intensity-dependent distortions such as thermal bending and blooming of these to occur more readily than
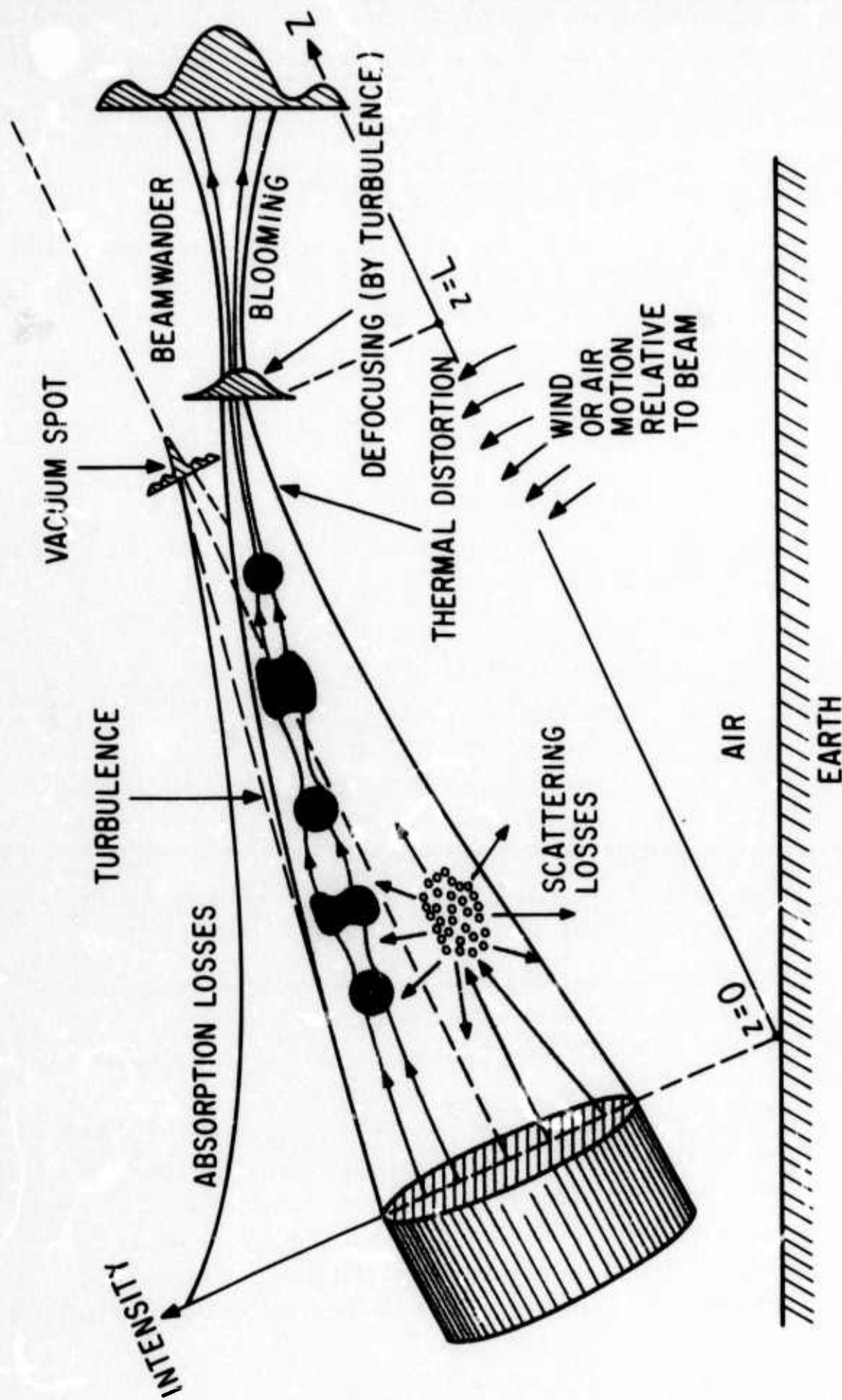
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Figure 1. Laser beam in air.

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of diverging beams. We shall see that the nature of turbulence-induced distortions is influenced importantly by beam shape. Also, the distortion of the image will or will not be important depending upon what aspects of the image are important in applications (tracking, weaponry, illumination, etc.), and consequently the study of which optical parameter should be considered [as indicated under (i)] is also of some importance.

A unified mathematical description of all of these aspects is based upon the concept of the electric field $E(\vec{r})$ at a location $\vec{r} = (\vec{\rho}, L)$ in the image plane. Under a number of not overly restrictive conditions (to be discussed later), it is given by

$$E(\vec{r}) = L^{-1} \exp[ik(L + \rho^2/2L)]$$

$$\times \frac{ik}{2\pi} \int d^2 \rho_1 B(\vec{r}, \vec{r}_1) U_o(\vec{r}_1) \exp[\frac{ik\rho_1^2}{2}] (\frac{1}{L} - \frac{1}{R}) - ik\vec{\rho} \cdot \vec{\rho}_1 / L] \quad , \quad (2.1)$$

where $\vec{r}_1 = (\vec{\rho}_1, 0)$ is a point on the aperture plane. The other symbols are:

$k = \omega/c = 2\pi/\lambda$: wavenumber of the laser radiation (ω = frequency, λ = wavelength).

L : propagation distance

R : radius of curvature of the phase front at $z = 0$ ($R > 0$ for a focused beam).

$U_o(\vec{r}_1)$: aperture or pupil function

$B(\vec{r}, \vec{r}_1)$: the ratio of the electric field at $\vec{r}_1$ due to a point source at $\vec{r}$ in air to that in free space.

The optical imaging properties we desire depend upon the use we make of Eq. (2.1). We may be interested in its phase properties (e.g., for angular deviations) or its intensity pattern (e.g., for illumination or for weapon uses). The nature of the beam is contained in the choice of R and of $U_o(\vec{r}_1)$. For an untruncated gaussian beam, one has $U_o(\vec{r}_1) = \exp(-\rho_1^2/r_o^2)$, where r_o^2 is an effective radius-variance parameter. For some applications it is more appropriate to take $U_o(\vec{r}_1) = $ const. for $\rho_1 \leq r_o$, and $U_o(\vec{r}_1) = 0$ for $\rho_1 > r_o$: this choice yields the well-known Airy diffraction pattern in the focal plane ($R = L$) of a focused beam. The most crucial aspect of the program - the interaction

of laser beams with (turbulent) air - is hidden in the function $B(\vec{r}, \vec{r}_1)$: the response to a spherically symmetric point source in turbulent air.

The distinction (i)-(iii) has been useful to the extent that it allows us to separate beam and imaging problems from the air-wave inter- action. It is a great simplification to study this interaction for only a spherical-wave source. Nevertheless, we will not adhere rigidly to this way of separating the problems; for some cases it will be easier to look at the interaction of air with beam waves.

Consider the basic interaction problem. A quasi-monochromatic point source at $\vec{r}_1 = 0$ yields an electric field E_o that satisfies the equation,

$$\vec{\nabla}^2 E_o + k^2 E_o = -\delta(\vec{r}) \quad , \tag{2.2}$$

where $\delta(\vec{r})$ is a Dirac-delta function. The free-space solution for E_o is (aside from some constant factors involving π) $E_o(\vec{r}) \sim r^{-1} \exp(ikr)$. In air we have

$$\vec{\nabla}^2 E + k^2 \epsilon(\vec{r}, t) E = -\delta(\vec{r}) \quad , \tag{2.3}$$

where $\epsilon(\vec{r}, t)$ is the dielectric permittivity. The equation is approxi- mate to the extent that it ignores terms of order $\partial\epsilon(\vec{r}, t)/\omega\partial t$ as well as terms of order $k|\nabla\epsilon(\vec{r}, t)|/\epsilon(\vec{r}, t)$. Optical spatial and temporal fre- quencies are so high that such approximations are warranted. Workers in optics often prefer to use the refractive index $n(\vec{r}, t)$ by substituting $\epsilon(\vec{r}, t) = n^2(\vec{r}, t)$ in Eq. (2.3). Obviously, the source function - a delta function in this case - can be eliminated by subtraction of Eq. (2.2) from Eq. (2.3), and the electric field E can be expressed in terms of E_o. If the source is at $\vec{r}_o$, then both E and E_o are functions of $\vec{r}$ and $\vec{r}_c$. The previously mentioned normalized field $B(\vec{r}, \vec{r}_o)$ is simply the ratio $E(\vec{r}, \vec{r}_o)/E_o(\vec{r}, \vec{r}_o)$.

It is possible to simplify Eqs. (2.2) and (2.3) by utilizing the ex- tremely small size of optical wavelengths with respect to scale sizes of variations in $\epsilon(\vec{r}, t)$. Under the so-called paraxial approximations, a parabolic equation for $B(\vec{r}, t)$ can be developed, namely

$$\nabla_T^2 B + 2ik(\hat{r} \cdot \vec{\nabla} - \frac{1}{r})B + k^2 \delta\epsilon B = 0 \tag{2.4}$$

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where $\vec{\nabla}_T$ is the gradient-operator component transverse to the radial direction, and $\delta\epsilon$ is the deviation of $\epsilon(\vec{r},t)$ from unity. This equation, and its plane-wave corollary, which differs from Eq. (2.4) to the extent that $(\hat{r}\cdot\vec{\nabla} - r^{-1})$ is replaced by $\partial/\partial z$, have been studied extensively by workers in propagation of waves through air.

The permittivity deviation $\delta\epsilon(\vec{r},t)$ in Eq. (2.4) is determined by the refractive index $n(\vec{r},t)$. In air, at optical frequencies, the latter is given by

$$n(\vec{r},t) = 1 + 10^{-6}(1 + 0.0075\lambda^{-2}) \times 77.6 \, p(\vec{r},t)/T(\vec{r},t) \qquad (2.5)$$

where pressure p is given in millibars, temperature T in degrees Kelvin, and wavelength λ in μm. The main cause of variations in $n(\vec{r},t)$ are those in the temperature. The so-called non-linear variations in T due to air heating by the laser beam itself are not the topic of this contract effort. Rather, the irregular fluctuations in T caused by turbulence - particularly of the boundary-layer type - are the subject of this work. Thus an atmospheric model for turbulence, possibly homogeneous turbulence, at altitude x is required. Tatarski [2] has elaborated on the connection between temperature and refractive-index statistics. Aside from constant factors one is given by the other. Consider therefore the Fourier transform of the covariance of the dielectric permittivity

$$\Phi(K) \equiv \int d^3\Delta r \langle \delta\epsilon(\vec{r},t)\delta\epsilon(\vec{r}+\Delta\vec{r},t)\rangle \exp(i\vec{K}\cdot\Delta\vec{r}) \qquad (2.6)$$

where $\vec{r}$ is a point at altitude x. The atmospheric-turbulence model specifies a spectrum,

$$\Phi(K) = 32\pi^3 \times 0.033 \, C_n^2(x)(K^2+L_o^{-2})^{-11/6}\exp(-K^2/\kappa_m^2) \quad , \qquad (2.7)$$

where macroscale L_o is presumably a function of x, microscale ℓ_o is related to κ_m by the relationship $\ell_o = 5.92\kappa_m^{-1}$, and $C_n^2(x)$ is the refractive-index structure constant related to its sea-level value [3] by the relationship

$$C_n^2(x) = C_n^2(x_o)(T_o/T)^2(x_o/x)^{2/3}(1 + x/\ell_s)^{-2/3}$$
$$\times \exp[-2(x-x_o)/h] \quad , \qquad (2.8)$$

where $x_o \simeq 1$ m and T_o are sea-level values of effective altitude and tempera-
ture, and $7\ell_s$ is the Monin-Obukhov length ($\ell_s \rightarrow \infty$ at dawn and dusk, and
$\ell_s \sim 1.5$ m at midday for well-developed turbulence). The structure func-
tion $C_n^2(x)$ can be obtained from temperature-variation measurements, and
the microscale κ_m^{-1}, although harder to estimate, enters into the calcu-
lations in such a manner that an error of a factor two or so in its mag-
nitude is not serious. The macroscale L_o varies more but it often drops
out of the end result.

These considerations, while not complete, give the essential inputs
to the calculations made under the auspices of this program. A previous
report TRVIII [1] gives a summary of results obtained up to February 1973.
Ensuing work will be discussed from here on.

3. GENERALIZATION OF THE FOCAL-SPOT BROADENING RESULT

Laser beams cannot focus into a point image because diffraction at the edges of the aperture through an angle $\theta_d \sim 1/kr_o$ yields a blurring at distance L of order $L\theta_d \sim L/kr_o$ (remember that r_o is a measure of the aperture radius). In turbulent air, the angle θ_d is randomly modified into $\theta_d + \delta\theta$, and $\delta\theta$ has a variance of the order of $C_n^2 L \kappa_m^{1/3}$ in homogeneously turbulent air (e.g., for horizontal propagation). The focal spot is broadened in area on the average.

In TRI, we defined an aperture radius for arbitrary aperture function $U_o(\vec{r}_1)$ - see Eq. (2.1) - through the definition

$$r_o^{-2} \equiv -\int d^2\rho_1 U_o \vec{\nabla}_T^2 U_o \Big/ \int d^2\rho U_o U_o \tag{3.1}$$

This definition corresponds to the one-sigma width of an untruncated gaussian aperture function, $U_o(\vec{r}_1) = \exp(-r_1^2/r_o^2)$. The integration and the gradient operator in Eq. (3.1) pertain to the two variables in the $z = 0$ plane. An irradiance-weighted radius r_L was defined in the plane $z = L$ by means of an integral,

$$r_L^2 = \int d^2\rho \rho^2 \langle I(\vec{\rho},L) \rangle \Big/ \int d^2\rho \langle I(\rho,L) \rangle \tag{3.2}$$

and a free-space equivalent, r_{Lo}, is defined similarly by utilizing $I_o(\vec{\rho},L)$ instead of $I(\vec{\rho},L)$. Actually, some first-moment integrals need to be subtracted, but they are zero for axially symmetric $U_o(\vec{\rho}_1)$ and for homogeneous turbulence. With these definitions, we obtained a very general result,

$$r_L^2 = r_{Lo}^2 + 12\pi^2 \times 0.033 L^2 \kappa_m^{1/3} \int_0^L ds \ s^2 C_n^2(s) \tag{3.3}$$

for an axial path from $s = 0$ to $s = L$. For horizontal propagation, $r_L^2 - r_o^2 \equiv r_{LB}^2 \simeq 1.3 C_n^2 L^3 \kappa_m^{1/3}$. Conditions for validity were given in TRI, and comparisons with other focal-area definitions were made in TRIV. The above definition requires care because r_{Lo}, hence also r_L, become infinite for sharply truncated beams. This difficulty is easily circumvented in practice by processing for $r_L^2 - r_{Lo}^2 \equiv r_{LB}^2$ (which is always finite) but the definition weights the side-lobe structure unduly. On

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the other hand, r_{LB}^2 has fundamental significance as the "displaced area" of the focal spot due to average turbulence broadening. It is given in Eq. (3.3) for the focal plane. We will now generalize it to any image plane.

Schmeltzer [4] has given the following form for the electric field of a gaussian laser mode in the plane z and close to the axis:

$$E_o(\vec{r}) = \frac{-ikw^2}{z-ikw^2} \exp[ikz + ik\rho^2/2(z-ikw^2)] \quad , \tag{3.4}$$

$$w^{-2} \equiv r_o^{-2} + ikR^{-1}$$

This expression is subject to

(i) $kz \gg 1$: radiation-zone condition

(ii) $z \ll kr_o^2$: near field of the aperture

$\left. \begin{array}{l} \text{(iii) } z^3 \gg kr_o^4 \\ \text{(iv) } z^3 \gg k\rho^4 \end{array} \right\}$: sagittal conditions

Conditions (i)-(iv) are sufficiently liberal to include all of the image in most cases when the propagation distance is at least a few hundred yards. It can be seen from the absolute square of $E_o(\vec{r})$, after straight-forward algebraic manipulation, that

$$I_o(\vec{r}) = (kr_o^2/z)^2 [1 + (1-z/R)^2 (kr_o^2/z)^2]^{-1}$$

$$\times \exp\{-(kr_o\rho/z)^2/[1 + (1-z/R)^2(kr_o^2/z)^2]\}$$

Thus, the free-space intensity at $\vec{r} = (\vec{\rho},z)$ is still a gaussian $\sim \exp(-\rho^2/r_o^2)$ where r_o is another constant radius determined from Eq. (3.5). The derivation of Eq. (3.3) from Eq. (3.2) is practically the same, except that we obtain $r_{Lo}^2(z)$ instead of r_{Lo}^2 to get the result: $r_L^2 = r_{Lo}^2(z) + r_{LB}^2$ with

$$r_{Lo}^2(z) = r_o^2[(1-z/R)^2 + (z/kr_o^2)^2] \quad , \tag{3.6}$$

where r_{LB}^2 is given by the second term of Eq. (3.3). This result is valid at any plane $z = $ const [subject to the conditions (i)-(iv)]. In that sense, it is more general than Eq. (3.3), but it is less general in the sense that a gaussian mode has been assumed for the aperture function

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$U_o(\vec{r})$. Note that Eq. (3.4) results from Eq. (2.1) upon inserting $U_o(\vec{r_1}) = \exp(-\rho_1^2/r_o^2)$ into the latter equation.

At $z = L = R$, we obtain from Eq. (3.6) the previous result. For collimated beams ($R \to \infty$), we note that diffraction is usually a small edge effect because $r_{Lo}^2(z) = r_o^2 + (z/kr_o)^2$ in that case, and because $z/kr_o^2 \ll 1$ so that $r_{Lo}(z) \simeq r_o$. Our previous studies (see Fig. 3 of TRVIII) have shown that $r_{LB} \ll r_o$ for most cases. Therefore, in the case of collimated beams - and certainly for diverging beams - the beam-broadening effect is less important. In those cases, amplitude scintillations are the dominant intensity effect. This immediately poses the question: do amplitude scintillations then also play an important role in the focal plane of a focused beam? Evidently, from Eqs. (3.3) and the preceding qualitative explanation on the basis of refractive bending of rays superimposed upon the free-space diffraction effect, phase effects are important. This question is answered in the next section.

4. BEAM WAVE IRRADIANCE AND PHASE IN TURBULENT AIR

We shall deal here with amplitude scintillation in the focal plane. Previously, we developed a comprehensive theory for plane waves (TRV and TRVI) and for spherical waves (TRVI). These theories are important for collimated and diverging beams respectively where (see previous section) we have shown that refractive (i.e., phase) effects are only important at the edge of the beam. We shall derive a Rytov approximation for gaussian-beam waves analogous to the development in sections 1 and 2 of TRV for plane waves and in section 2 of TRVII for spherical waves. It appears useful to give a very abbreviated review of the steps of the derivation. First of all we note that the normalized field B can be written as a sum,

$$B = 1 + B_1 + B_2 + B_3 + \cdots$$

$$B_n = \int d^3 r_1 G(\vec{r}, \vec{r}_1) \delta\varepsilon(\vec{r}_1) \cdots \int d^3 r_n G(\vec{r}_{n-1}, \vec{r}_n) \delta\varepsilon(\vec{r}_n) \quad , \tag{4.1}$$

where $G(\vec{r}, \vec{r}_1)$ is a Green's function for beam waves (all trivial factors can be absorbed in G). The next step consists of utilizing the small-angle scattering properties of $\delta\varepsilon$ which imply that $\vec{r}_{m-1} - \vec{r}_m$ makes only a small angle with the z axis. By means of a stationary-phase analysis (or a steepest-descent method one can perform all the transverse integrations in (4.1) to obtain formally something like

$$B_n = \int_0^L dz_1 f(z_1) \delta\varepsilon(0, z_1) \cdots \int_0^{z_{n-1}} dz_n f(z_n) \delta\varepsilon(0, z_n) g(z_1 \cdots z_n) \quad , \tag{4.2}$$

where $f(z_m)$ is a function resulting from (4.1) after the transverse integrations. The problem is that there is also an inseparable exponential function of $z_1 \cdots z_n$ which we denote by $g(z_1 \cdots z_n)$ that prevents us from separating the integrations over dz_m ($1 \le m \le n$) in general. The Rytov approximation is obtained by finding the conditions under which $g(z_1 \cdots z_n) = 1$. In that case (4.2) reduces to

$$B_n = \frac{1}{n!} \left[\int_0^L dz f(z) \, \delta\varepsilon(0, z) \right]^n \tag{4.3}$$

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and upon summation over n as in (4.1) we obtain

$$B = \exp[\int_0^L dz f(z) \delta\epsilon(0,z)] \tag{4.4}$$

We have abbreviated the actual procedure too much because it really has been carried out for moments of B rather than for B - thus yielding energy conservation which Eq. (4.4) does not yield - and we have omitted some intermediate extra steps such as utilizing transverse Fourier transforms and many statistical properties of $\delta\epsilon$, but the many details of the analysis are given in the above references (and exhaustively in TRIII).

Let us now give the details of this procedure for gaussian laser modes given by Eq. (3.4). Referring to section 2 of TRIII, we note that the beam-wave Green's function $G_b(\vec{r},\vec{r}_1)$ is given by

$$G_b(\vec{r},\vec{r}_1) = G_p(\vec{r}-\vec{r}_1)E_o(\vec{r}_1)/E_o(\vec{r}) \tag{4.5}$$

where E_o is given by Eq. (3.4), and $G_p(\vec{r}-\vec{r}_1)$ by Eq. (2.7) of TRIII. This Green's function is now germane in Eq. (3.1) of TRIII. In order to perform the $d^2\rho_m$ integrations in that equation, we perform a stationary-phase (steepest-descent) analysis on the integrand as in section 6 of TRIII. The stationary-phase function in the exponent of the integrand is

$$\psi_n = k[\Delta\rho_m^2 + \Delta z_m^2)^{1/2} + \rho_m^2/2Z_m - \rho_{m-1}^2/2Z_{m-1}]-\vec{K}_m\cdot\vec{\rho}_m$$
$$Z_m \equiv [z_m - ikw^2] \tag{4.6}$$
$$w^{-2} \equiv r_o^{-2} + ikR^{-1}$$

The points of stationary phase are found by setting the gradient of ψ_m with respect to $\vec{\rho}_m$ equal to zero. A result very close to the spherical-wave result of TRVII [Eq. (2)] is found:

$$\Delta\vec{\rho}_m/\Delta r_m = -\vec{Q}_m/k + \vec{\rho}_n/Z_n \tag{4.7}$$

The notation is consistent with previous work, and the interpretation of Eq. (4.7) is practically identical to that of Eq. (2) of TRVII. Again, small-angle scattering determines the physics, and small-angle

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approximations in Eq. (4.6) are permissible, yielding after some rearrangement:

$$\psi_m \simeq \frac{k}{2} \frac{z_m z_{m-1}}{z_{m-1} - z_m} \left(\frac{\vec{\rho}_{m-1}}{z_{m-1}} - \frac{\vec{\rho}_m}{z_m} \right)^2 - \vec{K}_m \cdot \vec{\rho}_m \tag{4.8}$$

This result is also quite parallel to that for spherical waves. From it, we derive, as in section 2 of TRVII but now for beam waves:

$$B_n = \prod_{j=1}^{n} \frac{ik}{8\pi^2} \int_0^{z_{j-1}} dz_j \int d^2K_j \delta\tilde{\epsilon}(\vec{K}_j, z_j)$$

$$\times \prod_{m=1}^{n} \exp[-iK_m^2(L-z_m)z_m/2kz_o]\exp[-i\vec{K}_m \cdot \vec{Q}_{m+1}'(L-z_m)/k] \tag{4.9}$$

$$\vec{Q}_{m+1}' \equiv \sum_{j=m+1}^{n} (z_j/z_o)\vec{K}_j$$

$$z_o \equiv L - ikw^2$$

We obtain the normalized field B from (4.9) by summing over all integer n from n = 0 on. Here too, as in section 17 of TRIII, the Rytov approximation (modified automatically so that energy is conserved) follows if the second exponential in (4.9) is very close to unity. In that case, the parameter $\langle \delta\chi^2 \rangle$ can be shown to be [in analogy to Eq. (17.10) of TRIII without the last exponential factor]:

$$\langle \delta\chi^2 \rangle = \frac{k^2}{16\pi} \int_0^L dz \int_0^\infty dKK\Phi(K)e^{-K^2\eta(z)(L-z)/k} \left\{ 1 - \cos[K^2\xi(z)(L-z)/k] \right\} \tag{4.10}$$

where we have utilized the real functions $\xi(z)$ and $\eta(z)$ from the definition,

$$\frac{z}{z_o} \equiv \frac{z - ikw^2}{L - ikw^2} \equiv \xi(z) - i\eta(z) \quad , \tag{4.11}$$

and where $\Phi(K)$ is given by Eq. (2.7). Rather than work out the condition for validity of (4.10) in general, we will now specialize to the case of a focused beam observed at its focal point, i.e., we choose R = L. It is important to note that (4.10) holds only for locations on the central axis anyway. Let us introduce the focusing factor $\beta_f = kr_o^2/L$, the ratio of the aperture radius to the radius of the diffraction-limited

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focal spot ($\beta_f \gg 1$). It can be seen from (4.11) that

$$\xi(z) = z/L \tag{4.12}$$
$$\eta(z) = \beta_f(L-z)/L \quad ,$$

in the focal plane. Set $\Phi(K) \simeq 32\pi\gamma C_n^2 K^{-11/3}$ where $\gamma = 0.033\pi^2 \simeq 0.326$ and substitute (4.12) into (4.10) to obtain

$$\langle \delta\chi^2 \rangle = 2 C_n^2 k^2 \int_0^\infty dK K^{-8/3} \int_0^L dz\, e^{-\beta_f K^2(L-z)^2/kL} \left\{ 1 - \cos[K^2 z(L-z)/kL] \right\} \tag{4.13}$$

Replace z by a new variable $z' = z/L$ (we leave off the prime for convenience) and let $x = K^2(L-z)/kL$. These two coordinate changes transform (4.13) into,

$$\langle \delta\chi^2 \rangle = \gamma C_n^2 k^{7/6} L^{11/6} \int_0^1 dz(1-z)^{5/6} \int_0^\infty dx\, x^{-11/6} e^{-\beta_f(1-z)x} [1-\cos(zx)]$$

$$= \gamma\Gamma(-5/6) C_n^2 k^{7/6} L^{11/6} \tag{4.14}$$

$$\times \int_0^1 dz(1-z)^{5/6} \left\{ [\beta_f(1-z)]^{5/6} - \mathrm{Re}[\beta_f(1-z) \pm iz]^{5/6} \right\}$$

This result is essentially equal to Eqs. (28) and (29) of Ishimaru [5]. For large β_f we utilize the asymptotic form for the braced factor in (4.14),

$$\{----\} \simeq -\frac{5}{72} \beta_f^{-7/6} z^2 (1-z)^{-7/6} \quad ,$$

to obtain a beta function in integral form, which yields:

$$\langle \delta\chi^2 \rangle = \frac{\gamma}{6} \cdot \frac{\Gamma(1/6)\Gamma(2/3)}{\Gamma(11/3)} \cdot \beta_f^{-7/6} C_n^2 k^{7/6} L^{11/6} \quad , \tag{4.15a}$$

The numerical coefficient in Eq. (4.15) is 0.102. The focusing factor β_f being quite large in all cases of interest, it follows that $\langle \delta\chi^2 \rangle$ is small compared to unity, and that is a sufficient condition for validity of the Rytov approximation. There is no practical need to work out an expression for $\langle \delta\chi^2 \rangle$ in the saturation regime because the log-amplitude variance appears to be small for all conceivable cases of interest. Let

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us insert $\beta_f = kr_o^2/L$ into Eq. (4.15a) to obtain,

$$\langle \delta\chi^2 \rangle \simeq 0.102 C_n^2 L^3 r_o^{-7/3} \quad , \tag{4.15b}$$

which is basically analogous to the geometrical-optics result for amplitude fluctuations in a turbulent medium (see page 251 of ref. 2) with scale-size cutoff at $\ell = r_o$. The result is not dependent upon wavelength.

The conclusion that $\langle \delta\chi^2 \rangle$ is very small for all practical situations ($r_o \sim 0.5m$, and L of the order of several kilometers) leads to an extremely important corollary: amplitude fluctuations are unimportant in the focal plane. It is therefore no coincidence that Eq. (3.3) can be obtained by a simple geometrical-optics ray-bending argument because refractive effects are dominant. To say it in yet another way, it appears sufficient to describe the distortion of the electric field on its way from the aperture to the focal plane by simple geometrical-optics phase fluctuations.

Let us now consider collimated beams, defined by allowing radius parameter R to become infinite in the definition of w^{-2} in (4.6). In that case we find from the new definition and from (4.11) that

$$\xi(z) = \frac{1 + zL/k^2 r_o^4}{1 + L^2/k^2 r_o^4} \simeq 1$$

$$\eta(z) = \frac{(L-z)/kr_o^2}{1 + L^2/k^2 r_o^4} \simeq (L-z)/kr_o^2 \tag{4.16}$$

When substituted into (4.10), $\eta(z)$ clearly yields a negligible attenuation factor. Therefore (4.10) reduces to the plane-wave form as we expect, provided $L \ll kr_o^2$ (i.e., provided edge diffraction is negligible).

Now consider diverging beams ($R < 0$) and particularly let $R \ll kr_o^2$. It follows from (4.6) and from (4.11) that

$$\xi(z) = (z+R)(L+R)/(L+R)^2 = (z+R)/(L+R)$$

$$\eta(z) = \frac{R}{kr_o^2} \cdot \frac{R(L-z)}{(L+R)^2} \ll 1 \tag{4.17}$$

Here again, $\eta(z)$ yields a negligible attenuation in (4.10), and now (4.10) reduces to the spherical-wave form for a point source radiating

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at the location $(0,0,-R)$. The condition $R \ll k r_o^2$ is not unduly restrictive.

In other words, the general beam-wave case is only of academic interest. Collimated and diverging beams can be treated, respectively, as plane and spherical waves with regard to amplitude scintillation. We have already performed a separate calculation for a focused beam which indicated that the saturation effect is not observable in the focal plane. This entire treatment of beam waves appears to be as exhaustive as is required for amplitude effects.

Some comments on phase are in order. In the model we have developed, there are diffractive and refractive effects. Diffractive effects are important for amplitude effects at all ranges; refractive effects start playing a role only in the saturation regime. With phase, refractive effects are important everywhere and it appears that diffractive ones can be ignored to a large extent. The most general expression we have for phase can be derived from Eq. (A4) of TRV, yielding

$$\delta\phi = (k/8\pi^2) \int_o^L dz \int d^2K \, \delta\tilde{\varepsilon}(\vec{K},z) e^{-i\vec{K}\cdot\vec{\rho}(z)} \cos[K^2(L-z)/2k] \qquad (4.18)$$

The last factor describes diffraction corrections which are important only from small eddies with size $\ell < (L/k)^{1/2}$, but on the other hand the phase variance is determined chiefly by the large eddies with $\ell \gg (L/k)^{1/2}$. Another way of showing the relative unimportance of the cosine factor in (4.18) is to determine the mean square of (4.18), ignoring the $\exp(-i\vec{K}\cdot\vec{\rho}(z))$ factor. After the usual universal approximations the cosine factors reduce to

$$\cos^2[K^2(L-z)/2k] = \frac{1}{2}\left\{1 + \cos[K^2(L-z)/k]\right\}$$

$$\qquad (4.19)$$

$$= 1 - \frac{1}{2}\left\{1 - \cos[K^2(L-z)/k]\right\}$$

This shows that $\langle(\delta\phi)^2\rangle$ reduces to $\langle(\delta\phi_g)^2\rangle - \langle\delta\chi^2\rangle$ where $\delta\phi_g$ is the phase (geometrical-optics) without the cosine factor. Our results indicate that $\langle\delta\chi^2\rangle \ll 1$, nearly everywhere except in the initial part of the saturation regime where $\langle\delta\chi^2\rangle$ can approach unity. Consequently $\langle(\delta\phi)^2\rangle \simeq \langle(\delta\phi_g)^2\rangle$.

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The significance of ray bending upon phase is easily illustrated from (4.18), ignoring the unimportant cosine factor, which can then be written,

$$\delta\phi = k\int_{0}^{s(L)} ds\,\delta\varepsilon(s), \tag{4.20}$$

where s is the ray-path parameter initiating at $s = 0$ and terminating at $s(L)$. We compare $\delta\phi$ to $\delta\phi_g$ given by a straight path:

$$\delta\phi_g = k\int_{0}^{L} dz\,\delta\varepsilon(0,0,z) \tag{4.21}$$

An estimate of the difference is given by

$$
\begin{aligned}
\delta\phi - \delta\phi_g &\simeq k\int_{0}^{L} dz\,[\,\delta\varepsilon(\vec{\rho}(z),z) - \delta\varepsilon(0,z)\,] \\
&\simeq k\vec{\rho}(z)\cdot\int_{0}^{L} dz\,\vec{\nabla}_T\,\delta\varepsilon(0,z) \\
&\simeq k\vec{\rho}(z)\cdot[d\vec{\rho}(z)/dz]_{z=L} \\
&\simeq \frac{1}{2}\,k\,[\frac{d\rho^2(z)}{dz}]_{z=L}
\end{aligned}
\tag{4.22}
$$

If we utilize the approximate relationship $\langle\rho^2(z)\rangle \sim C_n^2\kappa_m^{1/3}z^3$, we find that $\delta\phi - \delta\phi_g \sim C_n^2\kappa_m^{1/3}kL^2$. This is extremely interesting because the Rytov approximation has been shown to be valid when $C_n^2\kappa_m^{1/3}kL^2 \ll 1$. Thus the equivalence of $\delta\phi$ and $\delta\phi_g$, the unimportance of ray bending, is once again demonstrated as a sufficient condition for the Rytov approximation. A corollary of this statement is that ray bending must be considered in the saturation regime where (4.20) can differ appreciably from (4.21). In short, Eq. (4.20) is a non-trivial extension of (4.21), the usual phase approximation, into the saturation regime. The question of the statistics of $\delta\phi$ cannot be fully answered at this time because there are not enough measurements in the saturation regime. Buser and Born [11] have observed discontinuous phase jumps that seem to point towards the crossing of diverse ray bundles in the image plane. More measurements are needed to confirm this phenomenon which has been questioned.

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5. THE HERRMANN-BRADLEY NUMERICAL CALCULATION OF IRRADIANCE

Equation (2.4) for plane waves can be written as

$$\frac{\partial B}{\partial z} + (2ik)^{-1} \vec{\nabla}_T^2 B - \frac{ik}{2} \delta\varepsilon B = 0 \tag{5.1}$$

Our method of solving this equation for waves in turbulent air has been to convert it into an integral equation and to utilize small-angle approximations. An alternative method has been to set $B = \exp\psi$ and to derive an equation for ψ from (5.1). Our saturation-regime results were obtained by taking ray bending into account in the equation for ψ, i.e., by integrating along rays. Herrmann and Bradley [6] have attempted numerical integration of Eq. (5.1) using acceptable spatial-spectrum properties of $\delta\varepsilon$ to select samples of an ensemble. They then obtain laser-beam profiles of intensity that - ideally - should average to what can be calculated but have the advantage of giving as much detail as is desired (means and variances do not give much detail). A disadvantage of the method is that it costs time to produce a profile, and that it is perhaps difficult to produce enough to check the ensemble statistics.

The main question is whether the numerical solution is valid or whether the approximations made in generating it are too severe. We address ourselves to this question here.

Herrmann and Bradley convert (5.1) to an interaction representation by introducing the WKB phase increment

$$\delta\phi_g (\vec{\rho}, z) \equiv \frac{k}{2} \int_0^z dz_1 \delta\varepsilon (\vec{\rho}, z_1) \tag{5.2}$$

and by transforming B to $\tilde{B} = B\exp(-i\delta\phi)$. Introduce a new differential operator H,

$$H \equiv \frac{1}{2k} e^{-i\delta\phi_g} \vec{\nabla}_T^2 e^{i\delta\phi_g} \tag{5.3}$$

such that the Laplacian ∇_T^2 works not only upon $\exp(i\delta\phi_g)$ but also upon whatever function of $\vec{\rho}$ follows H within a terminating parenthesis. The transformation $B \rightarrow \tilde{B}$ and the definitions (5.2) and (5.3) then yield, when substituted into Eq. (5.1):

$$\partial\tilde{B}/\partial Z - i(H\tilde{B}) = 0 \tag{5.4}$$

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Now, a formal but not very useful way of generating a full solution of (5.4) is to divide both terms by $\tilde{B}$ and then integrate over z. This yields

$$\tilde{B}(z) = \exp\{-i \int_0^z dz_1 [\tilde{B}^{-1}(z_1) H \tilde{B}(z_1)]\}. \tag{5.5}$$

The crucial approximation made by Herrmann and Bradley in solving (5.5) is to replace $\exp(x)$ by $(1+x)/(1-x)$, where x is the exponent in (5.5). There are other approximations, but they are less critical. The ensuing implicit approximate equation in $\tilde{B}$ is then solved. If the approximation is to be a good one, we must demand that

$$\left| -i \int_0^z dz_1 [\tilde{B}^{-1}(z) H \tilde{B}(z_1)] \right| \ll 1 \quad , \tag{5.6}$$

and the error is to third order in this parameter. Transform $\tilde{B}$ back to B, and H back to $\vec{\nabla}_T^2$ to obtain

$$\left| \frac{-i}{2k} \int_0^z dz_1 B^{-1}(z_1) \vec{\nabla}_T^2 B(z_1) \right| \ll 1 \quad . \tag{5.7}$$

Now replace $\vec{\nabla}_T^2 B(z_1)$ in (5.7) by the other terms of Eq. (5.1) to obtain:

$$\left| \frac{ik}{2} \int_0^z dz_1 \delta\varepsilon(z_1) - \int_0^z dz_1 B^{-1}(z_1) \partial B(z_1)/\partial z_1 \right| \ll 1 \tag{5.8}$$

Replace B by $B = \exp(\chi + i\delta\phi)$ where χ is the log-amplitude and $\delta\phi$ the phase of the normalized field (both are real quantities), and utilize (5.2) to obtain

$$\left| i[\delta\phi_g - \delta\phi] + \chi \right| \ll 1 \tag{5.9}$$

Certainly both real and imaginary parts of (5.9) must be small separately in order for the inequality to hold. It follows that $\chi \ll 1$ is required. As χ is a stochastic quantity, perhaps normal in distribution (or at any rate not very differently distributed), it follows that its variance $\langle \delta\chi^2 \rangle$ must be much less than unity. This restricts the Herrmann-Bradley procedure to situations where the Rytov approximation is valid!

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We have shown in section 4 that amplitude fluctuations are small and properly described by the Rytov approximation in the focal plane. Thus it follows that the Herrmann-Bradley procedure may yield accurate results for focused laser beams. On the other hand, the procedure may be seriously in error when utilized to describe collimated and diverging beams under conditions under which the parameter combination $C_n^2 k^{7/6} L^{11/6}$ is not small.

6.   THE EFFECTS OF INTERMITTENCY

Most of the fluctuations of laser-beam parameters such as irradiance
and angle of arrival in turbulent air are directly due to fluctuations
in the refractive index. A major portion of these are due to temperature
fluctuations. The study of temperature fluctuations in turbulent air,
e.g., as discussed by Tatarski [2], yields the inputs to the analysis
of these fluctuating phenomena. Parameters such as the refractive-index
structure constant C_n^2 and the scales of turbulence ℓ_o (or κ_m^{-1}) and L_o
derive from the study of temperature fluctuations in turbulent air. In
recent years, workers at NOAA in Boulder, Colo. (R. S. Lawrence, G. Ochs,
et al.) in particular, but not exclusively, have pointed out certain
large-scale, low-frequency temperature fluctuations that have been ex-
cluded in prior treatments. These give rise to a class of phenomena that
do not appear to be sharply defined. Together with the underlying cause
these phenomena are referred to as "turbulence intermittency".

Recently, Kerr [7] has discussed some of the effects of intermittency.
He models the atmosphere into a number of irregular columns (slabs in
his specific treatment) with widely varying values of C_n^2, and computes
the effect upon the usual calculation of log-amplitude scintillation.
Seemingly separate from this model is a discussion of finite-time averag-
ing which can give rise to a data spread in apparently constant statis-
tical parameters such as means and averages.

It is not very clear, in our opinion, that the so-called intermittency
of turbulence can be defined and treated as some phenomenon separate from
what is otherwise just a random sampling of a statistical process with
known probability distribution and parameters. We will try to approach
a definition operationally as follows.

Out of a host of statistical parameters describing the behavior of
laser beams in turbulent air, certain variance parameters have been
chosen for analysis because they are relatively fundamental and easy to
measure. Two random variables, the variance of which we have analyzed,
are the log-amplitude and the angle of arrival. A common mode of des-
cription of both of these parameters as well as of others such as phase
be given in terms of integrals,

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$$\xi(t) = \frac{1}{4\pi^2} \int_0^L dz_1 \int d^2K \ f(\vec{K},z) \ \delta\tilde{\varepsilon}(\vec{K},z,t) \tag{6.1}$$

Here, $\delta\tilde{\varepsilon}(\vec{K},z,t)$ is the transverse two dimensional Fourier transform of the deviation in dielectric permittivity $\delta\varepsilon(\vec{r},t)$, and $f(\vec{K},z)$ is a determinate function. For example, if $\xi(t)$ is the log-amplitude, then $f(K,z)= -(k/2) \sin[K^2(L-z)/2k]$ in the Rytov approximation for plane waves. If $\vec{\xi}(t)$ is the angle of arrival vector, then $f(k,z)$ is the vector $-ik$. In order that $\xi(t)$ describes the Rytov phase we set $f(\vec{K},z)=(k/2) \cos[K^2(L-z)/2k]$. For the purpose of this section it does not matter what form $f(\vec{K},z)$ takes; Eq. (6.1) is sufficiently general for describing an operational definition of intermittency as it monifests itself in the optical phenomena.

Now define a mean square $C_\xi^2 \equiv \langle \xi^2 \rangle$; it is a variance as well as a mean square when $\langle \xi \rangle = 0$. However, we cannot measure C_ξ^2. What is really measured is a quantity,

$$C_\xi^2(t,T) \equiv \frac{1}{T} \int_{t-T/2}^{t+T/2} dt_1 \xi^2(t_1) \quad, \tag{6.2}$$

If (6.1) is substituted into (6.2), then the time-dependent part to scrutinize closely is

$$\frac{1}{T} \int_{t-T/2}^{t+T/2} dt' \delta\tilde{\varepsilon}(\vec{K}_1,z_1,t')\delta\varepsilon(\vec{K}_2 z_2,t') \tag{6.3}$$

Consider a random function $g(t)$, with zero mean for convenience. Let

$$g_T(t) = \frac{1}{T} \int_{t-T/2}^{t+T/2} dt' g(t') \quad. \tag{6.4}$$

and let $\langle g \rangle$ be the ensemble average of g. The temporal mean $g_T(t)$ will fluctuate around $\langle g \rangle$, and less so as T increases. If the statistics are stationary, then $\langle g \rangle$ will be the limit of $g_T(t)$ as $T \to \infty$, and it will indeed be independent of t. Hunt and Collins [8] have discussed this matter, and it is rather easy to reconstruct their result for the variance σ_T^2 of $g_T(t)$ around $\langle g \rangle$.

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$$\sigma_T{}^2 \sim (\tau/T)\sigma^2 \tag{6.5}$$

where σ^2 is the ensemble variance of $g(t)$, and τ is the integral time of the normalized auto-correlation of $g(t)$. Eq. (6.5) is correct only for $\tau \ll T$. In the opposite limit, $\sigma_T{}^2$ will vary randomly as $g^2(t)$ does because the frequency $2\pi T^{-1}$ lies beyond the frequencies of interest in the power spectrum of $g(t)$. When $\tau \ll T$, we may replace the time average by the ensemble average to good approximation.

Consider the time τ_0 to be the longer of L_0/U_T or $L_0/\delta U$ where U_T is the wind velocity component across the laser beam, and δU is the (more or less) isotropic root-mean-square wind velocity. Let averaging time T be much greater than τ_0. Our cardinal assumption for (6.3) is

$$\frac{1}{T} \int_{t-T/2}^{t+T/2} dt' \, \delta\widetilde{\varepsilon}(\vec{K}_1,z_1,t') \delta\widetilde{\varepsilon}(\vec{K}_2,z_2,t') \tag{6.6}$$

$$= \frac{4\pi^2}{T} \int_{t-T/2}^{t+T/2} dt' \, C_n{}^2(z_1,t') \Phi_2(\vec{K},\Delta z) \delta(\vec{K}_1+\vec{K}_2)$$

where $\Phi_2(\vec{K},\Delta z)$ is the partial Fourier transform of the autocovariance of $\delta\varepsilon(\vec{r})$ denoted as $F(K_x,K_y,\Delta z)$ by Tatarski [2], et al., and $\delta(\vec{K}_1+\vec{K}_2)$ is a two-dimensional Dirac delta function. The assumption implies also that the time variation in $C_n{}^2(z,t')$ is slow compared to the time τ_0 defined above. We now insert (6.6) into Eq. (6.2) and utilize the universal approximations that center around the rapid decay of $\Phi_2(\vec{K},\Delta z)$ with increasing Δz to obtain

$$C_\xi^2(t,T) = \frac{1}{4\pi^2} \int_0^L dz \int d^2K \, f^2(\vec{K},z) \, \Phi(K)$$

$$\times \frac{1}{T} \int_{t-T/2}^{t+T/2} dt' \, C_n{}^2(Z,t') \quad , \tag{6.7}$$

where $\Phi(K)$ is given by (2.6) and (2.7). The variance $C_\xi^2(t,T)$ is a fluctuating quantity to the extent that the last integral factor is. If $C_n^2(z,t')$ is time dependent, or - weaker statement- if the integral

$$\int_0^L dz\ C_n^2(z,t')\ f^2(\vec{K},z)$$

is a time varying quantity, then so is $C_\xi^2(t,T)$. Considering it established that $T \gg \tau$, we can see two possible causes of time variation:

(i) Non-stationary statistical behavior of $C_n^2(z,t')$ because T is so long that the heat input into the atmosphere changes (diurnal patterns).

(ii) Large-scale spatial variation in $C_n^2(z,t')$, associated with lengths L_c such that time L_c/U is not much smaller than T (obviously $L_c \gg L_o$). This second effect appears to be most basic to the concept of intermittency.

In order to proceed more specifically, we assume a large-scale frozen-flow model. That is to say, we assume that large-scale variations in $C_n^2(z,t')$ are blown across the line of sight by a wind with velocity vector $\vec{U}$ (unit vector $\hat{U}$) not necessarily perpendicular to the line of sight. The assumption implies at point $\vec{r}= (0,y,z)$:

$$C_n^2(\vec{r},t') = C_n^2(\vec{r}- \vec{U}(t'-t),\ t) \tag{6.8}$$

We will utilize this in (6.7). However, first we will assume a Kolmogorov spectrum derived from Eq. (2.7) for $\Phi(K)$ in (6.7) and formally perform the d^2K integration. Thus (6.7) becomes

$$C_\xi^2(t,T) = \int_0^L dz\ f(z)\ \frac{1}{T}\int_{t-T/2}^{t+T/2} dt'\ C_n^2(z,t') \tag{6.9}$$

The function $f(z)$ has the following form: For log-amplitude, $f(z) \sim 0.563\ k^{7/6}(L-z)^{5/6}$. For phase it is proportional to $k^2 L_o^{5/3}$, and for the focal-area parameter r_{LB}^2 it is $3.9\kappa_m^{1/3}(L-z)^2$. Similar and related

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forms for $f(z)$ result, depending on which parameter $\xi(t)$ is studied in Eq. (6.1). Now insert (6.8) into (6.9) and transform time variable t to spatial coordinate $s = Ut$ (s measures distance in the wind direction):

$$C_\xi^2(t,T) = \int_0^L dz f(z) \frac{1}{UT} \int_{-UT/2}^{UT/2} ds C_n^2(\vec{r} - s\hat{U}, t) \qquad (6.10)$$

The mean of (6.10) yields, under the assumption (for simplicity) that the mean of C_n^2 does not depend on $\vec{r}$,

$$\left\langle C_\xi^2 \right\rangle = \int_0^L dz f(z) \left\langle C_n^2 \right\rangle . \qquad (6.11)$$

In order to compute the variance of $C_\xi^2(t,T)$, we will introduce a spectrum $\phi_c(\vec{K})$ of the large-scale fluctuations of $C_n^2(\vec{r})$ via the transform

$$\left\langle C_n^2(\vec{r}_1) C_n^2(\vec{r}_2) \right\rangle - \left\langle C_n^2 \right\rangle^2$$
$$= \frac{1}{8\pi^3} \int d^3 K \phi_c(\vec{K}) \exp[-i\vec{K} \cdot (\vec{r}_1 - \vec{r}_2)] . \qquad (6.12)$$

Using (6.12), we can compute the variance of $C_\xi^2(t,T)$ from the average of the square of (6.10) minus the square of (6.11). The two resulting ds integrations are easily performed to obtain,

$$\left\langle C_\xi^4 \right\rangle - \left\langle C_\xi^2 \right\rangle^2 = \frac{1}{4\pi^2} \int d^2 K \phi_c(\vec{K})$$
$$\times \left| \frac{\sin(\vec{K} \cdot \vec{U}T/2)}{\vec{K} \cdot \vec{U}T/2} \int_0^L dz f(z) \exp(i\vec{K}, \vec{z}) \right|^2 \qquad (6.13)$$

The factor $\left| \sin(\vec{K} \cdot \vec{U}T/2)/(\vec{K} \cdot \vec{U}T/2) \right|^2$ acts as a low-pass filter particularly in directions normal to $\vec{z}$, upon the spectrum $\phi_c(\vec{K})$ of fluctuations in $C_n^2(z)$. It suppresses contributions from scale sizes $\ell \ll U_T T/2$ in analogy to the time-dependent case in (6.5).

Intermittency thus causes data spread in the variance parameter for relatively short averaging times T, and the variance of this data spread for a seemingly steady parameter C_ξ^2 (which would be constant at very

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large T under statistically homogeneous conditions) is given by Eq. (6.13). Consider as an example the log-amplitude $\xi = \ln A$ in which case C_ξ^2 is the log-amplitude variance. Then, Eq. (6.13) expresses the variance of the data spread of a finite-time log-amplitude variance in terms of the wind velocity U and the large-scale fluctuations of $C_n^2(z)$ that constitute $\Phi_c(\vec{K})$. We note that the data-spread variance decreases with increasing wind velocity when $U_T T/2$ is sufficiently large with respect to the important scale sizes. This trend is clearly visible in the log-amplitude variance measurements by Kleen and Ochs [9].

Intermittency, then, appears to be a large-scale variation in $C_n^2(z)$ as far as the optical effects are concerned. It affects the statistics of optical quantities such as phase and amplitude when these statistics are determined over time periods not very long compared to the time in which a typical large-scale variation crosses the beam. Its characteristics are determined by the (unknown) spectrum $\Phi_c(\vec{K})$ which in all likelihood is highly anisotropic and strongly characterized by local terrain and weather conditions.

The large-scale variations in refractive index probably have no direct optical consequences because both refractive and diffractive bending are governed by small-scale variations. An intermittent variation of statistical parameters such as C_n^2 and C_ξ^2 is brought about. Perhaps this variation should be regarded in the same way that a short-term (1-5 seconds) measurement is an instantaneous realization of which one can only give the statistics. Here, that would imply knowledge of (6.13) which in turn requires knowledge of $\Phi_c(\vec{K})$ which is probably not a universal function but a strong one of terrain and weather conditions locally. That may render attempts to determine $\Phi_c(\vec{K})$ in general ineffective a priori. In this sense it remains unclear whether intermittency phenomena can be distinguished from random sampling of a statistical process with known probability distribution.

7. ANGLE-OF-ARRIVAL SPECTRUM AND MICROSCALE

The power spectrum of the angle of arrival of a ray has been cal-
culated in TRIV, and an extension of this calculation for the angle-of-
arrival difference of two rays is given in TRVI. An important aspect
of this power spectrum (we consider it for one ray because the inter-
ferometer situation does not change matters significantly) is its high-
frequency behavior. Let $\dot{\omega}_T = U_T/L_o$ and $\Omega_T = \kappa_m U_T$ (we assume frozen flow
for the time being; the alternative situation has been discussed before).
The power spectrum $W_\theta(\omega)$ can then be written for $\omega \gg \omega_T$ as

$$W_\theta(\omega) = 4\gamma C_n^2 L L_o^{-1/3} W_\theta(\omega, 5/6),$$

$$W_\theta(\omega, 5/6) = \frac{\Gamma(1/3)}{\Gamma(5/6)} \frac{\sqrt{\pi}}{\omega_T} \left(\frac{\omega}{\omega_T}\right)^{-2/3} \qquad \text{for } \omega < \Omega_T \qquad (7.1)$$

$$= \frac{\sqrt{\pi}}{\omega_T} \left(\frac{\omega}{\omega_T}\right)^{-2/3} \left(\frac{\omega}{\Omega_T}\right)^{-1} e^{-(\omega/\Omega_T)^2} \qquad \text{for } \omega > \Omega_T \quad .$$

That is to say, there is a very sharp transition from an $\omega^{-2/3}$ dependence
for $\omega < \Omega_T$ to a steep gaussian decay $\sim \exp(-\omega^2/\Omega_T^2)$ when ω exceeds Ω_T.
The question arises whether this steep transition can be used to obtain
a microscale measurement by determining Ω_T under steady wind conditions
(so that κ_m may be retrieved from $\Omega_T = \kappa_m U_T$).

It appears to us that non-zero aperture limitations prevent an ob-
server from finding this transition, even in the idealized noise-free
case. In actual fact, the beam is not infinitesimally thin; it has a
non-zero thickness d (e.g., d is a gaussian halfwidth parameter of a
thin collimated beam). Temperature fluctuations at scale sizes ℓ larger
than d do indeed displace the entire beam (beam wander) as if it were a
ray, but smaller ones with $\ell < d$ decompose the beam into parts (beam
spreading). It is quite true that the sum of both effects would be
observed in long-term statistics to be governed by a variance
$\langle \theta^2 \rangle \sim C_n^2 L \ell_o^{-1/3}$ where ℓ_o is the microscale, but the measurement is
of the displacement of the center of irradiance, or of some related
quantity which is governed by a variance $\langle \theta^2 \rangle \sim C_n^2 L d^{-1/3}$. Tatarski [2]

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has computed a power spectrum for beam wander of a beam of thickness d, and he finds a spectrum that resembles (7.1) for $\omega < U/d$. However at $\omega \sim U/d$ the minus two-thirds behavior is terminated and a transition to a minus eight-thirds power of frequency occurs. The transition point is now characteristic of d rather than of ℓ. A second transition at $\omega \sim \Omega_T$ is no longer sharp and presumably hidden in noise.

The spectrum of (7.1) would indeed yield microscale information if aperture d were less than ℓ_o. Unfortunately ℓ_o can be very small (millimeter or less) indeed and the intensity of such narrow beams are probably too weak for use in this way. Therefore the limitations in extracting a microscale measurement from an angle-or-arrival spectrum appear too stringent at present.

8. MEAN FLUCTUATION RATES

The power spectrum of a fluctuating quantity contains much information about the time scales or the fluctuation rates of that quantity. For example, the power spectrum of a fluctuating amplitude of an electric field tells the energy of the field as a function of frequency. The integral of the power spectrum over all frequencies is simply the total energy (on the average) of the fluctuating field. A range of frequencies (fluctuation rates) ranging from U/L_o to U/ℓ_o roughly (where U is a typical velocity driving spatial irregularities across the laser beam) contributes to this total energy. One might wonder whether there is a dominant frequency. Perhaps inspection of the power spectrum might show up such a dominant frequency. However, it requires quite a bit of data processing to produce a power spectrum, and then it might be somewhat difficult to read a dominant frequency out of it. Furthermore, it is convenient to have a simpler more direct measure of the rate at which a random quantity, say the irradiance, fluctuates. A simple measure is the average number of zero crossings. The observer takes a certain time record of the irradiance, determines a mean irradiance level during that time, and then counts the number of times that the actual time record of irradiance crosses the mean level. It is clear, intuitively, that this count is a measure of the graininess, the irregularity of the fluctuating quantity.

Beckmann [10] gives the mean frequency of zero crossings as σ_f/π where the variance parameter σ_f^2 of the random quantity f is given by

$$\sigma_f^2 = - \frac{\partial^2}{\partial \tau^2}\left[\frac{\langle f(t)f(t+\tau)\rangle}{\langle f(t)f(t)\rangle}\right]_{\tau=0} \equiv - \frac{R_f''(0)}{R_f(0)} \tag{8.1}$$

We will compute this quantity for angle of arrival, amplitude, and phase.

Angle of Arrival - The autocorrelation of the angle of arrival of a ray is given by Eq. (31) of TRIV which is written equivalently as,

$$R_\theta(\tau) = 2\gamma C_n^2 L_o^{-1/3} \int_0^\infty dx\ x(1+x)^{-11/6} e^{-x/\kappa_m^2 L_o^2}$$
$$\times J_o(\omega_T \tau \sqrt{x}) e^{-x(\Delta\omega\tau/2)^2} \tag{8.2}$$

where $\gamma = 0.033\pi^2 \sim 0.326$, $\omega_T = U_T/L_o$, $\Delta\omega = 4\Delta U/L_o\sqrt{3}$.

This autocorrelation yields the variance for $\tau = 0$, namely $R_\theta(0) \sim 3.91\Gamma(7/6)C_n^2\kappa_m^{1/3}L$. The second derivative of the autocorrelation at $\tau = 0$, namely $R''(0)$ is easily computed from the second line in Eq. (8.2) which yields

$$\frac{\partial^2}{\partial\tau^2}\left[J_o(\omega_T\tau\sqrt{x})e^{-x(\Delta\omega\tau/2)^2}\right]_{\tau=0} \tag{8.3}$$

$$= -\frac{x}{2}(\omega_T^2+\Delta\omega^2) \equiv -\frac{x}{2}(v^2/L_o^2)$$

where $v^2 = U_T^2 + 16\Delta U^2/3$ is a variance of velocity that combines the frozen-flow and the random-flow velocities into a single parameter. Unless $U_T \approx 0$, the difference between v and U_T is negligible. Using (8.3) in (8.2) we apply the definition (8.1) for angle of arrival $(f = \theta)$ to obtain

$$\sigma_\theta^2 = \frac{v^2}{2L_o^2}\int_o^\infty dx\, x^2(1+x)^{-11/6}e^{-x/\kappa_m^2 L_o^2}\bigg/\int_o^\infty dx\, x(1+x)^{-11/6}e^{-x/\kappa_m^2 L_o^2} \tag{8.4}$$

In numerator as well as in denominator, it is permissible to replace $1+x$ by x because $\kappa_m L_o \gg 1$. It is then easily established that

$$\sigma_\theta^2 = \frac{\pi^2}{3}(v/\ell_o)^2 \quad . \tag{8.5}$$

Thus a non-surprising result has been found in yet one other way: the angle-of-arrival fluctuations are governed by the smallest turbulence-scale sizes. In practice (see section 7) the graininess will probably not be determined by ℓ_o but by the diameter d of a pencil beam by which a ray is simulated. Thus, the fluctuation rate v/ℓ_o or v/d is a measure of the most important rate of angular beam deviations.

Log-amplitude - The log-amplitude variance in the Rytov approximation is given by

$$\left\langle \delta\chi^2 \right\rangle = \gamma C_n^2 k^{7/6}L^{11/6}\int_o^\infty dx\, x^{-11/6}(1-x^{-1}\sin x) \tag{8.6}$$

in which expression the variable x was obtained from the wavenumber variable K through the transformation $x = K^2 L/k$. As usual, $\gamma = 0.033\pi^2 \sim 0.326$. We generalize $\langle \delta\chi^2 \rangle = \langle \delta\chi(t)\delta\chi(t) \rangle$ to $R_A(\tau) = \langle \delta\chi(t+\tau)\delta\chi(t) \rangle$ by the same procedure as in the case of angle-of-arrival, i.e., we replace $\Phi(K)$ by

$$\Phi(K)J_o(KU_T\tau)\exp(-\frac{4}{3}\,K\Delta U^2\tau^2) \quad , \tag{8.7}$$

(see Eq. (30) of TRIV). After transforming K to x, and utilizing the frequency variables ω_T and $\Delta\omega$, we insert the extra factors in (8.7) into (8.6) to obtain

$$
R_A(\tau) = \gamma C_n^2 k^{7/6} L^{11/6} \int_0^\infty dx\, x^{-11/6}(1-x^{-1}\sin x)e^{-kx/\kappa_m^2 L}
$$
$$
\times\; J_o(\omega_T\tau\sqrt{y})e^{-y(\Delta\omega\tau/2)^2} \tag{8.8}
$$

where $y \equiv x(kL_o^2/L)$. The calculation of $\sigma_A^2 = -R_A''(0)/R_A(0)$ is easily repeated by utilizing the intermediate step (8.3) to obtain,

$$
\sigma_A^2 = \frac{kv^2}{2L}\int_0^\infty dx\, x^{-5/6}(1-\frac{\sin x}{x})e^{-kx/\kappa_m^2 L} \Big/ \int_0^\infty dx\, x^{-11/6}(1-\frac{\sin x}{x})e^{-kx/\kappa_m^2 L}
\tag{8.9}
$$

The integral in the denominator can be approximated well by ignoring the exponential factor. It then yields the factor $\pi/[2\Gamma(17/6)\sin(7\pi/12)]$ in the denominator. The integral in the numerator can be approximated by ignoring the sinx/x term, and it yields a factor $\Gamma(1/6)(\kappa_m^2 L/k)^{1/6}$. The result is

$$
\sigma_A^2 = \frac{\Gamma(1/6)\Gamma(17/6)\sin(7\pi/12)}{\pi}\left(\frac{\kappa_m^2 L}{k}\right)^{1/6}(kv^2/L)
$$
$$
\sim 2.95(\kappa_m^2/k)^{1/6}(kv^2/L)
\tag{8.10}
$$

Bearing in mind that the factor $(\kappa_m^2 L/k)^{1/6}$ is of order unity in optical propagation problems of interest, we note that the rate of zero crossings is given by the frequency $v(k/L)^{1/2}$, i.e., by those irregularities with scale size $(L/k)^{1/2}$. That is in good agreement of course with the

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interpretation of the spatial covariance of log-amplitude which is also governed by a length $\sim (L/k)^{1/2}$.

Phase - Finally, the calculation is easily repeated for the phase increment which we write

$$\delta\phi = \frac{1}{2}k \int_0^L dz \delta\epsilon(z,t) \quad , \tag{8.11}$$

where a somewhat unimportant filter factor has been ignored. Define $R_\phi(\tau) = \langle \delta\phi(t)\delta\phi(t+\tau)\rangle$. We obtain it from $R_\phi(0)$ by means of the substitution (8.7) and get

$$R_\phi(\tau) = 4\gamma C_n^2 k^2 L \int_0^\infty dK K (K^2 + L_o^{-2})^{-11/6} e^{-K^2/\kappa_m^2} J_0(KU_T\tau)\exp[-4(K\Delta U\tau)^2/3]$$

$$= 2\gamma C_n^2 k^2 L L_o^{5/3} \int_0^\infty dx(1+x)^{-11/6} J_0(\omega_T\tau\sqrt{x}) e^{-x[(\Delta\omega\tau/2)^2 + 1/\kappa_m^2 L_o^2]} \tag{8.12}$$

Note that this yields the well-known phase variance $R_\phi(0) = \langle \delta\phi^2 \rangle = (12/5)\gamma C_n^2 k^2 L L_o^{5/3}$. Utilizing (8.3) we obtain

$$R_\phi''(0) = \gamma C_n^2 k^2 L L_o^{-1/3} v^2 \int_0^\infty dx\, x(1+x)^{-11/6} e^{-x/\kappa_m^2 L_o^2} \quad , \tag{8.13a}$$

which can be approximated well by replacing $(1+x)$ by x to yield,

$$R_\phi''(0) \sim \gamma\Gamma(1/6)(\kappa_m L_o)^{1/3} C_n^2 k^2 L L_o^{-1/3} v^2 \quad . \tag{8.13b}$$

Thus we find for the zero-crossing rate parameter,

$$\sigma_\phi^2 = \frac{5}{12}\Gamma(1/6)(\kappa_m^{1/3} v^2/L_o^{5/3}) \quad , \tag{8.14}$$

which result indicates that phase fluctuations appear to be governed by a rate $\sim v/(L_o^{5/6}\ell_o^{1/6})$, which is not very different from v/L_o because $(\ell_o/L_o)^{1/6}$ is of order unity.

The calculations of zero crossings thus seem to indicate that angle of arrival, amplitude, and phase each have their own typical fluctuation rate f namely,

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angle of arrival : $f_\theta \sim v/\ell_o$

log-amplitude   : $f_A \sim v/(L/k)^{1/2}$

phase   : $f_\phi \sim v/L_o^{5/6} \ell_o^{1/6} \sim v/L_o$

The full power spectrum indicates a spread of rates between v/L_o and v/ℓ_o (there is an artificial smoothing of the spectrum between $\omega = 0$ and $\omega \sim v/L_o$ that has no physical meaning) but these typical rates are dominant in the sense that the average number of surges and fades of signal records are given by them.

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